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REFLECTION AND TRANSMISSION OF SHEAR WAVES IN MONOCLINIC MEDIA

A. CHATTOPADHYAY[1,*], S. SAHA[2] AND M. CHAKRABORTY[1]

[1] *Department of Applied Mathematics, Indian School of Mines, Dhanbad, 826 004, India*
[2] *Central Road Research Institute, New Delhi 110 020, India*

SUMMARY

This paper deals with the propagation, reflection and transmission of shear waves in monoclinic media. The dispersion equation for a monoclinic layer overlying a monoclinic half-space has been obtained and curves are plotted. The amplitude ratios for both the reflected and transmitted waves due to reflection of shear waves at the interface of two monoclinic half-spaces have also been computed and the numerical results are presented graphically. The results are compared with the isotropic case. It has been observed that, in monoclinic media, the amplitude ratios for reflected and transmitted wave increases approximately by 25 and 50 per cent respectively, in comparison to the isotropic case. © 1997 by John Wiley & Sons, Ltd.

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INTRODUCTION

Wave propagation in crystalline media plays a very interesting role in Geophysics and also in ultrasonics and signal processing. The studies of propagation, reflection and transmission of waves are of great interest to seismologists. Such studies help them to obtain knowledge about the rock structure, as well as the elastic properties and at the same time information regarding minerals and fluids present inside the earth.

The problem of reflection and refraction of elastic waves in anisotropic layered media has been studied by many authors. Without going into details, some of the papers which may be cited are those in References 1–9. Wave propagation in crystalline monoclinic plates was studied by Chattopadhyay and Bandyopadhyay[10] and in multilayered anisotropic media by Nayfeh.[11]

The exterior of the earth is made up of solids, liquids and occluded gases. The solids are commonly called rocks. When minerals occur with definite geometrical outlines they are called crystals. Crystals are solids bound by natural plane surfaces or faces. A variety of crystal forms is possible. The monoclinic form is one of them. Monoclinic systems correspond to crystals which can be referred to three unequal axes, two of which intersect at an oblique angle, while the third is perpendicular to the other two.[12] Rotated y-cut quartz, which exhibits monoclinic symmetry, is formed by rotation of the thickness direction around the x-axis. y-cut crystals are used to generate

* Correspondence to A. Chattopadhyay, Department of Applied Mathematics, Indian School of Mines, Dhanbad, 826 004, India

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shear vibration in solids.[13] Seismic waves of high energy can produce structural change in the mineral constituents of the rocks through which they are propagated. Seismic wave velocities are usually greater in crystalline rocks than in sedimentary rocks.

In this paper, we discuss the problem of propagation, reflection and transmission of shear waves in a monoclinic medium. In Sections 2 and 3 of the paper we study the propagation of shear waves in a layered monoclinic medium in which the upper medium is a layer of thickness H and the lower one is a half space. The dispersion curves for the monoclinic medium are compared with those for an isotropic medium. It is found that as the value of dimensionless wave number increases the value of the dimensionless phase velocity decreases. The numerical results have been presented graphically. In Sections 4 and 5 of the paper we consider the problem of reflection and transmission of shear waves at the interface of two monoclinic media. The numerical values of the amplitude ratios of both reflected and transmitted waves have been computed for the monoclinic media and compared with the corresponding values of isotropic media. For the range of angle of incidence from 0 to 80°, the value of the amplitude ratio of the reflected wave increases. The maximum difference occurs at 10° and exhibits about a 25 per cent increase in comparison to the isiotropic case. The value of the amplitude ratio of the transmitted wave increases in the range 0–90° and the difference is maximum at an angle of incidence of 10° with about a 50% increase in comparison to the isotropic case.

2. FORMULATION OF THE PROBLEM

In this section the propagation of shear waves in a layered monoclinic medium overlying a monoclinic half space is studied.

The strain displacement relations are

$$S_1 = \frac{\partial u}{\partial x}, \qquad S_2 = \frac{\partial v}{\partial y}$$

$$S_3 = \frac{\partial w}{\partial z}, \qquad S_4 = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \tag{1}$$

$$S_5 = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \qquad S_6 = \frac{\partial y}{\partial x} + \frac{\partial u}{\partial y}$$

where u, v, w are displacement components in the directions x, y, and z, respectively and S_i ($i = 1, 2, \ldots, 6$) are the strain components. The stress–strain relations for rotating y-cut quartz which exhibits monoclinic symmetry with x being the diagonal axis are [14]

$$T_1 = C_{11}S_1 + C_{12}S_2 + C_{13}S_3 + C_{14}S_4$$

$$T_2 = C_{21}S_1 + C_{22}S_2 + C_{23}S_3 + C_{24}S_4$$

$$T_3 = C_{31}S_1 + C_{32}S_2 + C_{33}S_3 + C_{34}S_4$$

$$T_4 = C_{41}S_1 + C_{42}S_2 + C_{43}S_3 + C_{44}S_4$$

$$T_5 = C_{55}S_5 + C_{56}S_6$$

$$T_6 = C_{65}S_5 + C_{66}S_6 \tag{2}$$

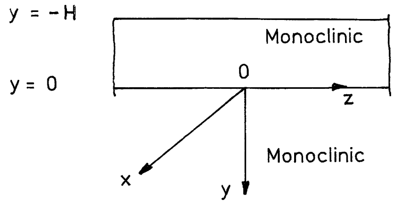

Figure 1. Geometry of the problem

where T_i ($i = 1, 2, \ldots, 6$) are stress components and C_{ij} ($i = 1, 2, \ldots, 6$; $j = 1, 2, \ldots, 6$) are elastic constants.

For waves propagating in the z-direction and causing displacements in the x-direction only (Figure 1), we shall assume that

$$v = w = 0 \quad \text{and} \quad u = u(y, z, t). \tag{3}$$

The equations of motion without body forces are

$$\begin{aligned} \frac{\partial T_1}{\partial x} + \frac{\partial T_6}{\partial y} + \frac{\partial T_5}{\partial z} &= \rho \frac{\partial^2 u}{\partial t^2} \\ \frac{\partial T_6}{\partial x} + \frac{\partial T_2}{\partial y} + \frac{\partial T_4}{\partial z} &= \rho \frac{\partial^2 v}{\partial t^2} \\ \frac{\partial T_5}{\partial x} + \frac{\partial T_4}{\partial y} + \frac{\partial T_3}{\partial z} &= \rho \frac{\partial^2 w}{\partial t^2} \end{aligned} \tag{4}$$

where ρ is the density of the medium.

Using (1) and (3) relations (2) become

$$\begin{aligned} T_1 = T_2 = T_3 = T_4 &= 0 \\ T_5 &= C_{55}\frac{\partial u}{\partial z} + C_{56}\frac{\partial u}{\partial y} \\ T_6 &= C_{56}\frac{\partial u}{\partial z} + C_{66}\frac{\partial u}{\partial y} \end{aligned} \tag{5}$$

3. SOLUTION OF THE PROBLEM

Inserting (5) in (4), the equations of motion for the upper and lower media in the absence of body forces are, respectively,

$$C_{66}\frac{\partial^2 u}{\partial y^2} + 2C_{56}\frac{\partial^2 u}{\partial y\,\partial z} + C_{55}\frac{\partial^2 u}{\partial z^2} = \rho\frac{\partial^2 u}{\partial t^2} \tag{6}$$

$$C'_{66}\frac{\partial^2 u_1}{\partial y^2} + 2C'_{56}\frac{\partial^2 u_1}{\partial y\,\partial z} + C'_{55}\frac{\partial^2 u_1}{\partial z^2} = \rho'\frac{\partial^2 u_1}{\partial t^2} \tag{7}$$

where u, u_1 are the displacement components in the x-direction, C_{ij}, C'_{ij} are the elastic constants and ρ, ρ' are the densities for the upper layer of thickness H and the lower half-space, respectively.

The Boundary conditions are as follows: the shear stress vanishes at the free surface and at the welded contact, the displacements and tractions are continuous, i.e.,

$$\begin{gathered} T_6 = 0 \quad \text{at } y = -H \\ u = u_1 \quad \text{at } y = 0 \\ (T_6)_1 = (T_6)_2 \quad \text{at } y = 0 \end{gathered} \tag{7a}$$

It is assumed that for a wave propagating along the positive z-direction in the upper medium

$$u = u(y)\exp[-\mathrm{i}(\omega t - kz)] \tag{8}$$

where k is the wave number and ω ($=kc$) is the circular frequency in which c is the phase velocity of the wave.

Substituting (8) in (6), we get

$$\frac{\mathrm{d}^2 u}{\mathrm{d}y^2} + \alpha\frac{\mathrm{d}u}{\mathrm{d}y} + \beta u = 0 \tag{9}$$

where

$$\alpha = 2\mathrm{i}k\frac{C_{56}}{C_{66}}, \quad \beta = \frac{\rho\omega^2 - C_{55}k^2}{C_{66}} \tag{10}$$

Hence, the solution of (6) is

$$u(y, z, t) = \mathrm{e}^{-(\alpha/2)y}(A\cos py + B\sin py)\mathrm{e}^{-\mathrm{i}(\omega t - kz)} \tag{11}$$

where

$$p = \sqrt{\frac{\rho\omega^2 - k^2 C_{55}}{C_{66}} + k^2\frac{C_{56}^2}{C_{66}^2}}$$

The solution of equation (7), for the lower half-space is

$$u_1(y, z, t) = A_1\mathrm{e}^{-(\alpha'/2 - 1/2\sqrt{\alpha'^2 - 4\beta'})}\mathrm{e}^{-\mathrm{i}(\omega t - kz)} \tag{12}$$

where

$$\alpha' = 2\mathrm{i}k\frac{C'_{56}}{C'_{66}}, \qquad \beta' = (\rho'\omega^2 - C'_{55}k^2)/C'_{66}$$

Using (11) and (12) in the boundary conditions (7a) and eliminating A, B, and A_1 we get the dispersion equation as

$$\tan\left[kH\left\{\left(\frac{C_{56}}{C_{66}}\right)^2 + \frac{c^2}{(C_{66}/\rho)} - \left(\frac{C_{55}}{C_{66}}\right)\right\}^{1/2}\right]$$
$$= \frac{C'_{66}}{C_{66}}\frac{\left[\frac{C'_{55}}{C'_{66}} - \left(\frac{C'_{56}}{C'_{66}}\right)^2 - \frac{c^2}{(C'_{66}/\rho')}\right]^{1/2}}{\left[\left(\frac{C_{55}}{C_{66}}\right)^2 + \frac{c^2}{(C_{66}/\rho)} - \left(\frac{C_{55}}{C_{66}}\right)\right]^{1/2}} \tag{13}$$

For the real solution of equation (13), we have

$$\sqrt{\frac{C_{55}}{C_{66}}-\left(\frac{C_{56}}{C_{66}}\right)^2}<\frac{c}{\beta_1}<\frac{\beta_2}{\beta_1}\sqrt{\frac{C'_{55}}{C'_{66}}-\left(\frac{C'_{56}}{C'_{66}}\right)^2}$$

where $\beta_1=[C_{66}/\rho]^{1/2}$ and $\beta_2=[C'_{66}/\rho']^{1/2}$ are the shear wave velocities in the upper and lower media.

3.1. *Numerical calculations and discussions*

For numerical calculations, the following values of the elastic constants have been used[14]

$$C_{55}=0{\cdot}94\ \mathrm{N/m^2},\qquad C_{66}=0{\cdot}93\ \mathrm{N/m^2}$$

$$C_{56}=-0{\cdot}11\ \mathrm{N/m^2},\qquad \rho=7450\ \mathrm{kg/m^3}$$

$$C'_{55}=0{\cdot}60\ \mathrm{N/m^2},\qquad C'_{56}=0{\cdot}09\ \mathrm{N/m^2}$$

$$C'_{66}=0{\cdot}75\ \mathrm{N/m^2},\qquad \rho'=4700\ \mathrm{kg/m^3}$$

$$\beta_1/\beta_2=0{\cdot}88$$

It has been noted that the dimensionless phase velocity (c/β_1) which is a function of the dimensionless wave number, lies between 0·9984 and 1·0018 which are the lower and upper bounds, respectively, in case of monoclinic media. For a homogeneous and isotropic medium, on the other hand the dimensionless phase velocity lies between 1·001 and 1·21. Hence, the effect of monoclinic media is distinctly marked. For different values of the phase velocity c/β_1, curves have been plotted against kH (Figure 2) and compared to those for the isotropic case. From the graphs, it appears that the dimensionless phase velocity increases faster for the isotropic case than for monoclinic media. It is observed that the c/β_1 ratio is approximately 20 per cent lower in the monoclinic case than in the isotropic case for a value of $kH=5$.

4. FORMULATION AND SOLUTION

In this section, the reflection and transmission of shear waves at the interface of two half-spaces of monoclinic type is studied.

Let the two half-spaces be in contact along the plane $y=0$ and the y-axis be positive downwards in the lower half-space. A shear wave incident at the interface from the lower half-space will give rise to a reflected shear wave in the same half-space and a transmitted shear wave in the upper half-space. Let the angles at the interface made by the incident and the transmitted wave be e and f, respectively, (Figure 3).

The equation of motion will be the same as (6) and (7). The solution of (6) and (7) for the upper and lower half-spaces may be written as

$$u(y)=A\mathrm{e}^{-\alpha y/2}\mathrm{e}^{\eta y}\mathrm{e}^{-\mathrm{i}(\omega t-kz)} \tag{14}$$

$$u_1(y)=(A_1\mathrm{e}^{\eta_1 y}+B_1\mathrm{e}^{-\eta_1 y})\mathrm{e}^{-\alpha' y/2}\mathrm{e}^{-\mathrm{i}(\omega t-kz)} \tag{15}$$

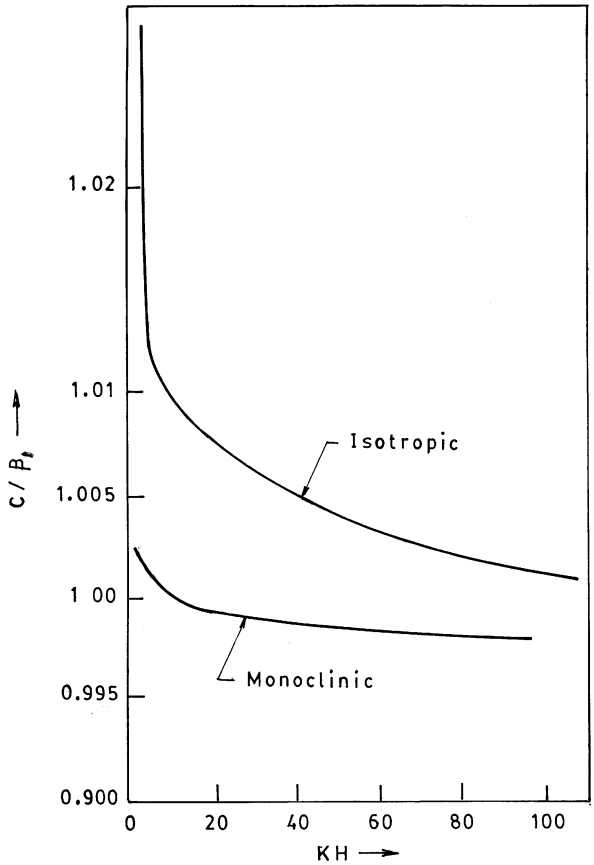

Figure 2. Variation in C/β_1 with respect to kH for monoclinic medium

where

$$\alpha = 2ik\frac{C_{56}}{C_{66}}, \qquad \beta = \frac{C_{55}k^2 - \rho\omega^2}{C_{66}}$$

$$\eta = \frac{1}{2}\sqrt{\alpha^2 + 4\beta}, \qquad \alpha' = 2ik\cdot\frac{C'_{66}}{C'_{66}}$$

$$\beta' = \frac{C'_{55}k^2 - \rho'\omega^2}{C'_{66}}, \qquad \eta_1 = \frac{1}{2}\sqrt{\alpha'^2 + 4\beta'}$$

The boundary conditions at the interface are

$$u = u_1 \quad \text{at } y = 0 \tag{16}$$

$$(T_6)_1 = T_6 \quad \text{at } y = 0 \tag{17}$$

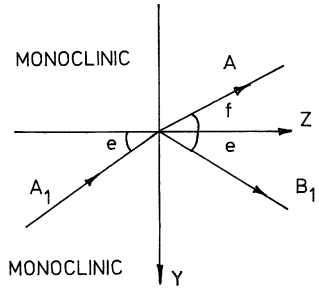

Figure 3. Reflection and refraction of shear waves

Using the boundary conditions (16) and (17) in (14) and (15), the amplitude ratios of reflected (B_1/A_1) and refracted (A/A_1) waves may be calculated as

$$\frac{B_1}{A_1} = \frac{1-M}{1+M} \tag{18}$$

$$\frac{A}{A_1} = \frac{2}{1+M} \tag{19}$$

where

$$M = \frac{C_{66}}{C'_{66}} \frac{\left[\frac{C_{55}}{C_{66}} - \left(\frac{C_{56}}{C_{66}}\right)^2 - \frac{c^2}{(C_{66}/\rho)}\right]^{1/2}}{\left[\frac{C'_{55}}{C'_{66}} - \left(\frac{C'_{56}}{C'_{66}}\right)^2 - \frac{c^2}{C'_{66}/\rho}\right]^{1/2}} \tag{20}$$

$$\beta_1^2 = C_{66}/\rho, \qquad \beta_2^2 = C'_{66}/\rho'$$

Snell's law relating the reflected and the transmitted waves is

$$c = \beta_2 \sec e, \qquad c = \beta_1 \sec f, \tag{21}$$

$$\tan e = \left(\frac{c^2}{\beta_2^2} - 1\right)^{1/2}, \qquad \tan f = \left(\frac{c^2}{\beta_1^2} - 1\right)^{1/2}$$

On substitution of (21) in (20), we have

$$M = \left(\frac{C_{66}}{C'_{66}}\right) \frac{\left[\frac{C_{55}}{C_{66}} - \frac{C_{56}}{C_{66}} - \frac{\beta_2^2}{\beta_1^2}(1+\tan^2 e)\right]^{1/2}}{\left[\frac{C'_{55}}{C'_{66}} - \left(\frac{C'_{56}}{C'_{66}}\right)^2 - (1+\tan^2 e)\right]^{1/2}}$$

The wave is completely transmitted when

$$\frac{A}{A_1} = 1$$

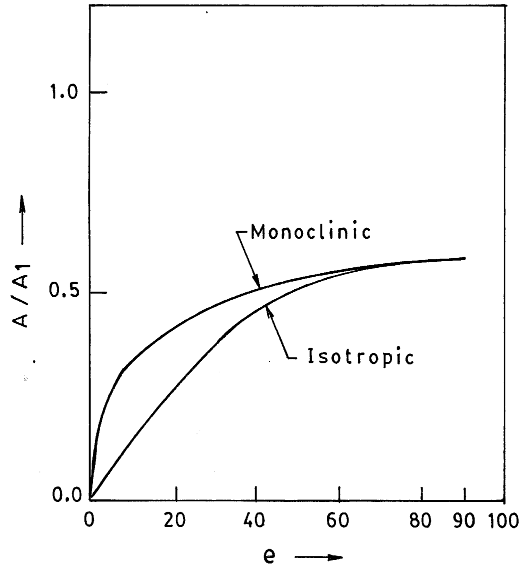

Figure 4. Variation of A/A_1 with respect to e

i.e.,

$$e = \tan^{-1}\left[\frac{\dfrac{C_{66}}{C'_{66}}\left\{\dfrac{C_{55}}{C_{66}} - \dfrac{C'_{55}}{C_{66}} + \dfrac{C'^2_{56}}{C_{66}C'_{66}} - \dfrac{C^2_{56}}{C^2_{66}} + \dfrac{C'_{66}}{C_{66}} - \left(\dfrac{\beta_2}{\beta_1}\right)^2\right\}}{\dfrac{C_{66}}{C'_{66}}(\beta_2/\beta_1)^2}\right]^{1/2}$$

4.1. Numerical calculations and discussions

Using the same values of the parameters considered in the previous case (6) and taking $(\beta_2/\beta_1)^2 = 3{\cdot}459$, the amplitude ratios for the refracted (A/A_1) and the reflected (B_1/A_1) waves have been computed and curves are plotted for different values of the angle of incidence ranging from 0 to 90° (Figures 4 and 5). The value of A/A_1 for the monoclinic medium increases faster in the range 0–70° and is always larger than for the isotropic case (Figure 4). In the monoclinic medium the rate of increase from 0 to 10° is very high and from 10 to 70° it is slow and uniform. The difference in values of A/A_1 for the monoclinic medium and the isotropic case is maximum at $e = 10°$. It is approximately 1.5 times more than the isotropic case. Similar nature of the curves (Figure 5) are observed for B_1/A_1. The maximum difference in the values of B_1/A_1 for the monoclinic medium and isotropic case occurs at the angle of incidence $e = 10°$. It is approximately 1.25 times more in comparison to the isotropic case.

5. CONCLUDING REMARKS

From the above studies it can be concluded that in a monoclinic medium the velocity of the shear waves is significantly different to that in an isotropic medium. The magnitudes of the reflection

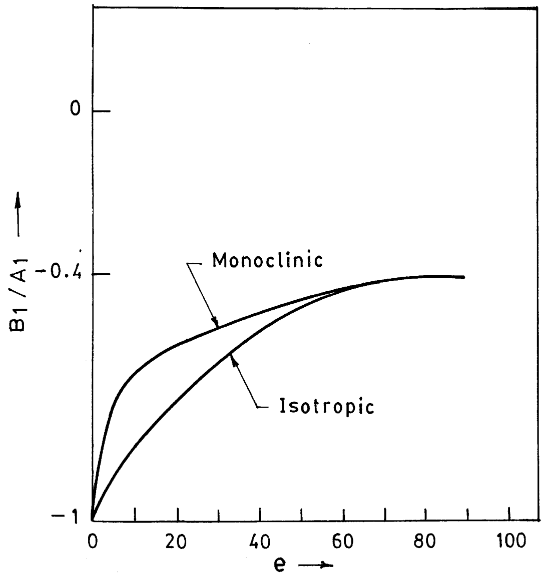

Figure 5. Variation of B_1/A_1 with respect to e

and the transmission coefficients of shear waves in a monoclinic medium are also remarkably different to those in an isotropic medium. The amplitude ratios of reflected and transmitted shear waves in monoclinic media are having much higher values and, therefore, the results can be utilized in the interpretation and analysis of data of Geophysical studies. The findings will be helpful in forecasting formation details at greater depth through signal processing and seismic data analysis. The present study may be effectively utilized to generate initial data prior to exploitation of the formation. This study may be useful to Geophysicist and Metallurgists for analysis of rock and material structures through Non-Destructive Testing (NDT).

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